

Family Name: _____ Given Name: _____ I.D.# _____

MAT3320 Assignment 3

Total: 10 marks. Due date: Tuesday, June 27, on or before 4:00pm.

In MATH Department (585 King Edward), there is a Drop-Box. You need to put your assignment into the box **on or before 4:00pm** on the due date. Late assignments will not be accepted.

1. (3 marks) Let $f(x) = \begin{cases} 1, & 0 \leq x < 1; \\ 3 - x, & 1 \leq x \leq 2. \end{cases}$. The Fourier sine series of $f(x)$ is

$$FSS(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right).$$

- (i) (2 marks) Find b_6 .
 (ii) (1 mark) Find $FSS(2015)$.
2. (4 points) Consider the wave equation $16u_{tt} = u_{xx}$, subject to the boundary conditions $u(0, t) = u(3, t) = 0$ and the initial conditions $u(x, 0) = 0, u_t(x, 0) = 3\sin(6\pi x) - 2\sin(9\pi x)$. The solution is

$$u(x, t) = \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi ct}{L}\right) + b_n \sin\left(\frac{n\pi ct}{L}\right) \right] \sin\left(\frac{n\pi x}{L}\right).$$

Find a_n, b_n and the detail solution.

3. (3 points) Solve $u_{xx} = \frac{1}{4}u_t, 0 < x < 2, t > 0$, subject to the boundary conditions $u(0, t) = 0, u(2, t) = 6$, and the initial condition $u(x, 0) = 3\sin(3\pi x) - 2\sin\left(\frac{7\pi x}{2}\right) + 3x$.

(Hint: Let $u(x, t) = v(x) + w(x, t)$ where $v(x)$ is a linear function such that $w(0, t) = w(2, t) = 0$.)